JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 10, No. 1, January–March 1996

# Technical Comments

## Comment on "Corrective Term in Wall Slip Equations for Knudsen Layer"

Roop N. Gupta*
*NASA Langley Research Center, Hampton, Virginia 23681*
and
Carl D. Scott†
*NASA Johnson Space Flight Center, Houston, Texas 77058*

IN a recent Technical Note,[1] Goniak and Duffa obtained an additional term to the surface temperature slip equation of Ref. 2. This term has been referred to as a corrective term in Ref. 1. However, the derivation of this additional term should be viewed from the degree of approximation involved and the validity[3] of continuum methods to treat transition regime. The so-called correction term is, in fact, a higher-order term and may not be significant in the part of transition regime where a continuum flow analysis can be used, i.e., for $Kn < 1$ and Knudsen layer thickness of order mean free path. In their analysis, Gupta et al.[2] considered only the random thermal velocity and not the mean flow (or drift) velocity (which was considered negligible across the Knudsen layer). This is consistent with the approaches of Shidlovskiy[4] and Patterson,[5] who neglected the convection (or slip) velocity at the edge of Knudsen layer in their velocity distribution functions. Shidlovskiy assumed a nondrifting Maxwellian distribution function, whereas Patterson used a nondrifting Maxwellian with Hermite polynomial perturbation terms. In Ref. 2, a nondrifting Chapman–Enskog velocity distribution function was assumed. The additional (or correction) term of Ref. 1 results from the inclusion of mean flow (or drift) velocity in the analysis. The temperature slip equation with this additional term is[1]

$$T_s = T_w - [(2-\theta)/\theta](\tfrac{1}{2}RT)^{1/2}\sqrt{\pi}(T_s/P_s)q_y + \underline{(1/4R)u_s^2}$$

or, in the nondimensional form

$$\bar{T}_s = \bar{T}_w - \mathcal{O}(\varepsilon^2) + \mathcal{O}(\varepsilon^4)$$

with

$$\varepsilon^2 = \mu_{\text{ref}}/\rho_\infty U_\infty r_n$$

as defined in Ref. 2. Obviously, the additional term (underlined above) is of higher order as compared to the second term on the right-hand side. In fact, one should retain other terms of order $\varepsilon^2$ [see Eq. (73b) of Ref. 2] before this term is considered important in the temperature slip equation. Further, $u_s$ must be near 100 m/s to increase the slip temperature $T_s$ by 10 K through this additional term.[1]

Thus, there is no correction involved in the temperature slip equation of Ref. 2 for a formulation of order $\varepsilon^2$. For flow conditions, where terms of order $\varepsilon^4$ become important, a continuum approach to analyze transition flow regime may be called in question.[3]

### References

[1] Goniak, R., and Duffa, G., "Corrective Term in Wall Slip Equations for Knudsen Layer," *Journal of Thermophysics and Heat Transfer*, Vol. 9, No. 2, 1995, pp. 383, 384.
[2] Gupta, R. N., Scott, C. D., and Moss, J. N., "Slip-Boundary Equations for Multicomponent Air Flow," NASA TP-2452, Nov. 1985; see also "Surface Slip Equations for Low-Reynolds Number Multicomponent Gas Flows," AIAA Paper 84-1732, June 1984.
[3] Bird, G. A., *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, Clarendon, Oxford, England, UK, 1994, p. 4.
[4] Shidlovskiy, V. P., *Introduction to the Dynamics of Rarefied Gases*, Elsevier, New York, 1967, pp. 60–68.
[5] Patterson, G. N., *Molecular Flows of Gases*, Wiley, New York, 1956.

Received July 7, 1995; accepted for publication Sept. 18, 1995. 
*Senior Aerospace Engineer, Aerothermodynamics Branch. Associate Fellow AIAA.
†Senior Scientist, Aeroscience Branch. Associate Fellow AIAA.

## Reply by the Author to R. N. Gupta and C. D. Scott

G. Duffa*
*Commissariat à l'Energie Atomique, 33114 Le Barp, France*

IN a Technical Note,[1] Goniak and Duffa have given an expression for the temperature slip in a Knudsen layer. This expression contains an additive term compared to the previous work of Grad[2] and Gupta et al.[3]:

$$T_s = T_w - \frac{2-\theta}{2\theta}\cdot\frac{\beta_s T_s \pi^{1/2} q_y}{P_s} + \frac{u_s^2}{4R} \tag{1}$$

This expression is based on the solution of a problem in which the Knudsen layer is supposed collision-free. This hypothesis implies constant half fluxes of conservative quantities: mass, momentum, and energy. This second item was given by Grad without physical justification.

It was demonstrated that theoretical inconsistencies or differences in previous works are resolved by the last term in Eq. (1). This term is proportional to $(\beta_w u_s)^2$, the squared value of the slip velocity $u_s$ divided by the most probable value $1/\beta_w$ of the velocity of a Maxwellian distribution at wall. This term scales with $1/\rho_w^2$ and then seems a priori to be small.[4,5]

Received Sept. 6, 1995; accepted for publication Sept. 18, 1995. 
*Senior Scientist, CEA/CESTA, BP2.

**Table 1 Effect of the corrective term in Eq. (1)**

| | Without the last term in Eq. (1) | With the last term in Eq. (1) |
|---|---|---|
| $T_s - T_w$ | 961 K | 910 K |
| $u_s$ | 196 m/s | 194 m/s |

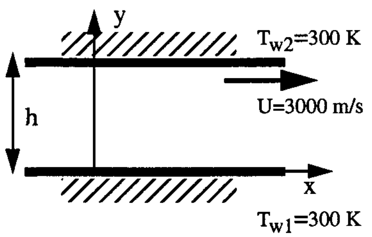


**Fig. 1 Couette problem.**

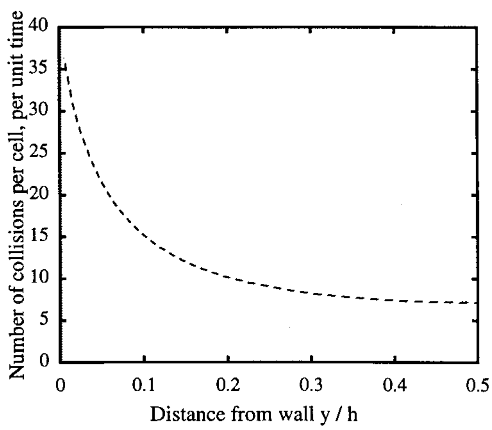


**Fig. 2 Number of collisions.**

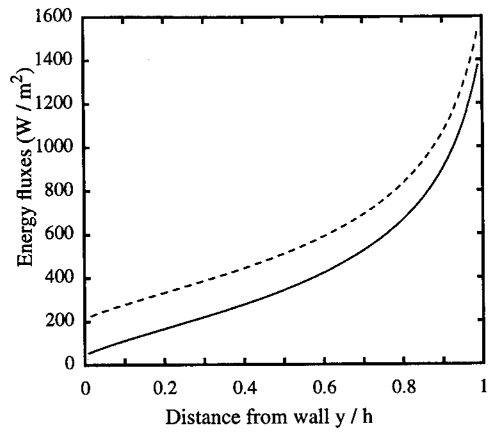


**Fig. 3 Up and down energy fluxes.**

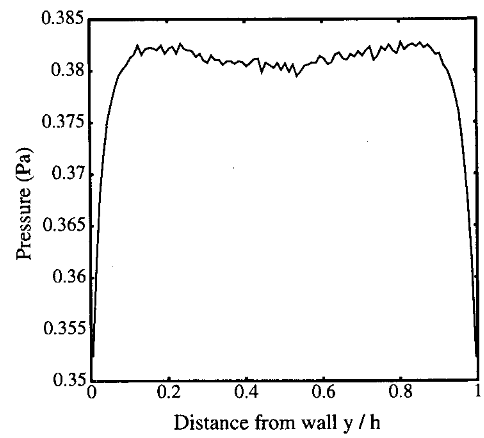


**Fig. 4 Pressure.**

A numerical calculation on an academic test-case, the Couette problem (Fig. 1), can illustrate this point. For example,[5] in a moderately rarefied situation with a Knudsen number of ~0.05 (local value based on $h$), we get the following results (Table 1), which give some idea on the magnitude of this last correcting term.

This academic situation is interesting because it is monodimensional and then permits accurate direct simulation Monte Carlo (DSMC) calculations.

These calculations demonstrate some interesting features:

1) The region near the wall is where the collisions number is maximum (Fig. 2).

2) Consequently, the fluxes are not constant (Fig. 3).

3) The sole macroscopic variable for which the Knudsen layer is clearly visible is the pressure. This fact seems to be confirmed in the work of Le Tallec[6] and can be explained as a consequence of the presence of non-Chapman–Enskog distributions near the wall (Fig. 4).

4) Even if the Knudsen layer thickness is difficult to define precisely in the DSMC results, it is clear that the slips are very different from those given by the Navier–Stokes equations with slip wall conditions, whatever correcting term is plugged in. A DSMC calculation will actually predict larger jumps:

$$T_s - T_w \approx 1050 \text{ K} \qquad u_s \approx 400 \text{ m/s}$$

Fortunately, the differences on other interesting values are smaller, e.g., 13% on the heat flux.

In conclusion, one can say that even if the work in the Technical Note[1] solves an apparent second-order discrepancy of previous work on this subject, the actual solution of Knudsen layer slip problem is not satisfactory since the basic hypothesis made by a classical slip continuum approach on the analytic solutions is not satisfied and the numerical predictions differ from a kinetic one, the difference being larger if one adds the correcting term. This perfectly agrees with the comment of Gupta and Scott.

## References

[1]Goniak, R., and Duffa, G., "Corrective Term in Wall Slip Equations for Knudsen Layer," *Journal of Thermophysics and Heat Transfer*, Vol. 9, No. 2, 1995, pp. 383, 384.

[2]Grad, H., "On the Kinetic Theory of Rarefied Gases," *Communications on Pure and Applied Mathematics*, Vol. 2, No. 4, 1949, pp. 331–407.

[3]Gupta, R. N., Scott, C. D., and Moss, J. N., "Surface Slip Equations for Low-Reynolds Number Multicomponent Gas Flow," AIAA Paper 84-1732, June 1984.

[4]Gupta, R. N., and Scott, C. D., "Comment on 'Corrective Term in Wall Slip Equations for Knudsen Layer,'" *Journal of Thermophysics and Heat Transfer*, Vol. 10, No. 1, 1996, p. 190.

[5]Goniak, R., "Partially Rarefied Flows near a Wall," Ph.D. Dissertation, Provence Aix-Marseille Univ., France, June 1994 (in French).

[6]Bourgat, J. F., and Le Tallec, P., "Transitional Boundary Layers Kinetic Studies," INRIA/MENUSIN Rept. G-93-13117, Aug. 1994 (in French).